

Local Maxwell symmetry and gravity

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In this paper we discuss some physical aspects of a theory obtained by gauging the AdS-Maxwell symmetry. Such theory has the form of Einstein gravity coupled to the $\text{SO}(3,1)$ Yang-Mills field. We notice that there is another tetrad field, which can be associated with linear combination of Lorentz and Maxwell connections. Taking this tetrad as a fundamental variable makes it possible to cast the theory into the form of $f - g$ gravity, in first order formulation. Finally we discuss a simple cosmological model derived from the AdS-Maxwell gravity.

I. INTRODUCTION

The Maxwell symmetry is an extension of Poincaré symmetry, being a symmetry of fields on constant electromagnetic background [1], [2]. This symmetry is of interest from purely algebraic point of view because it circumvents a well known theorem that does not allow for central extension of this algebra (see e.g., [3], [4], [5]).

However, until recently the Maxwell symmetry did not attract much interest. This is somehow surprising, because physical systems living on constant electromagnetic background, or closely to such a situation related, are frequently encountered and important in physics. This suggests that the Maxwell symmetry may play a quite important role in physics. For example since the canonical non-commutativity (for review see [6]) is ultimately related to the behavior of quantum systems on constant electromagnetic background (or generalization thereof), one can expect that this symmetry may play an important role in non-commutative field theories (as already noticed in [7].) By the same token it does not seem excluded that it might be relevant in the context of quantum Hall effect.

Leaving these speculations to future works, in this paper we investigate the related problem, namely the construction and properties of theories with local Maxwell invariance. In the next section, following [13], we recall the formulation of such a theory, deriving also its field equations. The following short section is devoted to some comments on possible relations between the gauged Maxwell theories and bi-metric theories of gravity. Finally, in section IV we discuss a simple class of cosmological solutions of this theory.

II. GAUGED MAXWELL THEORY

We begin this section presenting some definitions and then recalling some of the results presented in [13].

The starting point of our construction is a generalization of Maxwell symmetry, in which the Poincaré subalgebra of the original Maxwell algebra [1], [2] is replaced with the anti de Sitter counterpart. In other words we consider the algebra of symmetries of (complex scalar, Dirac) fields on anti de Sitter space and constant electromagnetic background¹. The algebra of such symmetries, which turns out to be isomorphic to $\mathfrak{so}(3,1) \oplus \mathfrak{so}(3,2)$ was discussed previously in [10], [11], [12] and has the following form [13]

$$\begin{aligned} [\mathcal{P}_a, \mathcal{P}_b] &= i(\mathcal{M}_{ab} - \mathcal{Z}_{ab}), \\ [\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= -i(\eta_{ac}\mathcal{M}_{bd} + \dots), \\ [\mathcal{M}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \dots), \\ [\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \dots), \\ [\mathcal{M}_{ab}, \mathcal{P}_c] &= -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \quad [\mathcal{Z}_{ab}, \mathcal{P}_c] = 0, \end{aligned} \quad (2.1)$$

where \dots denotes three more terms obtained by anti-symmetrization in the pairs of indices (ab) and (cd) . The original Maxwell algebra [10], [11], [12] can be easily obtained from (2.1) by contraction, which makes \mathcal{M} on the right hand side of the first commutator and the whole right hand side of the fourth one vanish.

This algebra can be gauged by defining the gauge field (connection)

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \frac{1}{2}h_\mu^{ab}\mathcal{Z}_{ab}, \quad (2.2)$$

and its curvature

$$\mathbb{F}_{\mu\nu} = \partial_\mu\mathbb{A}_\nu - \partial_\nu\mathbb{A}_\mu - i[\mathbb{A}_\mu, \mathbb{A}_\nu], \quad (2.3)$$

which can be decomposed into Lorentz, translational, and Maxwell parts

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}T_{\mu\nu}^a\mathcal{P}_a + \frac{1}{2}G_{\mu\nu}^{ab}\mathcal{Z}_{ab}, \quad (2.4)$$

where

$$F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} + \frac{1}{\ell^2}(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b), \quad (2.5)$$

$$T_{\mu\nu}^a = D_\mu^\omega e_\nu^a - D_\nu^\omega e_\mu^a, \quad (2.6)$$

$$\begin{aligned} G_{\mu\nu}^{ab} &= D_\mu^\omega h_\nu^{ab} - D_\nu^\omega h_\mu^{ab} - \frac{1}{\ell^2}(e_\mu^a e_\nu^b - e_\nu^a e_\mu^b) \\ &\quad + (h_\mu^{ac} h_\nu^{cb} - h_\nu^{ac} h_\mu^{cb}). \end{aligned} \quad (2.7)$$

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¹ A detailed discussion will be presented in a forthcoming paper.

In the formula above we denote by

$$D_\mu^\omega(*) \equiv \partial_\mu(*) - i[\omega_\mu, (*)]$$

the covariant derivative of the Lorentz connection ω .

The gauge transformations of the gauge fields h_μ^{ab} , ω_μ^{ab} , e_μ^a can be calculated to be

$$\delta_\Theta h_\mu^{ab} = D_\mu^\omega \tau^{ab} + h_\mu^{ac}(\lambda + \tau)_c^b + h_\mu^{bc}(\lambda + \tau)_c^a \quad (2.8)$$

$$\delta_\Theta \omega_\mu^{ab} = D_\mu^\omega \lambda^{ab} \quad (2.9)$$

$$\delta_\Theta e_\mu^a = -\lambda_b^a e_\mu^b, \quad (2.10)$$

where λ^{ab} and τ^{ab} denote the parameter of local Lorentz and Maxwell symmetries, respectively. The components of the curvature transform in a homogeneous way

$$\delta_\Theta G_{\mu\nu}^{ab} = -[\tau, F_{\mu\nu}]^{ab} - [(\lambda + \tau), G_{\mu\nu}]^{ab} \quad (2.11)$$

$$\delta_\Theta F_{\mu\nu}^{ab} = -[\lambda, F_{\mu\nu}]^{ab} \quad (2.12)$$

$$\delta_\Theta T_{\mu\nu}^a = -\lambda_b^a T_{\mu\nu}^b. \quad (2.13)$$

In the formulas above we omitted local translations, which, as usual, are being eventually traded for general coordinate invariance and are not symmetries of the action.

It is clear from eqs. (2.10), (2.11)–(2.13) that the only geometrical action i.e., a Lorentz and Maxwell invariant four-form that can be built from gauge fields and curvatures (see [13] and references therein) has the form

$$S_E = \frac{1}{64\pi G} \int \epsilon_{abcd} \left(R_{\mu\nu}^{ab} e_\rho^c e_\sigma^d - \frac{\Lambda}{3} e_\mu^a e_\nu^b e_\rho^c e_\sigma^d \right) \epsilon^{\mu\nu\rho\sigma} \quad (2.14)$$

being the standard Einstein action with a cosmological term. Of course, there are other possible invariant terms, which are quadratic in the curvatures; these terms are however topological and do not influence equations of motion.

Let us comment at this point the relation between our construction and that presented in the recent paper [9]. In that paper the authors construct the curvatures similar to the ones we presented above (2.5)–(2.7), but then, to build the action, they use all possible geometric combinations that are local Lorentz invariant. In this way the theory considered in [9] breaks the Maxwell symmetry manifestly. For this reason the geometric action derived in [9] is much richer than the one derived in [13] and presented above.

The action (2.14) does not include curvature of the Maxwell field G (as said above all the geometrical terms including G are the topological ones) and one wonders if one could find nontrivial terms which include the Maxwell field strength. Inspecting (2.11), (2.12) it is easy to see that there are only two possible terms of this sort. The first is the Yang-Mills action with the $\text{SO}(3, 1)$ gauge group, already found in [4], [13]

$$S_1 = -\frac{1}{4\varrho_1} \int d^4x e (F_{\mu\nu}^{ab} + G_{\mu\nu}^{ab})(F^{\mu\nu}_{ab} + G^{\mu\nu}_{ab}). \quad (2.15)$$

Noting that ϵ^{abcd} is an invariant tensor of $\text{SO}(3, 1)$, we can construct yet another manifestly gauge invariant action

$$S_2 = -\frac{1}{4\varrho_2} \int d^4x e (F_{\mu\nu}^{ab} + G_{\mu\nu}^{ab})(F^{\mu\nu cd} + G^{\mu\nu cd}) \epsilon_{abcd}. \quad (2.16)$$

In the formulas above ϱ_1 and ϱ_2 are two dimensionless coupling constants.

Using the explicit form of the curvatures F and G , (2.5) and (2.7), the form of these actions can be further simplified. Indeed

$$F_{\mu\nu}^{ab}(\omega, e) + G_{\mu\nu}^{ab}(h, \omega, e) = H_{\mu\nu}^{ab}(\varpi)$$

where $H_{\mu\nu}^{ab}(\varpi)$ is the curvature tensor of the connection $\varpi_\mu^{ab} \equiv \omega_\mu^{ab} + h_\mu^{ab}$. From now on we consider ϖ to be the new fundamental variable. The total action is the sum of the Einstein one (2.14) and the actions for the field ϖ

$$\begin{aligned} S &= S_E + S_1 + S_2 \\ &= \frac{1}{64\pi G} \int \epsilon_{abcd} \left(R_{\mu\nu}^{ab} e_\rho^c e_\sigma^d - \frac{\Lambda}{3} e_\mu^a e_\nu^b e_\rho^c e_\sigma^d \right) \epsilon^{\mu\nu\rho\sigma} \\ &\quad - \frac{1}{4} \int e \left(\frac{1}{\varrho_1} H_{\mu\nu}^{ab} H^{\mu\nu}_{ab} + \frac{1}{\varrho_2} H_{\mu\nu}^{ab} H^{\mu\nu cd} \epsilon_{abcd} \right). \end{aligned} \quad (2.17)$$

The field equations resulting from this action are Einstein equations (with cosmological constant term)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (2.18)$$

whose right hand side is the energy momentum tensor for the field H

$$\begin{aligned} T_{\mu\nu} &= \frac{1}{\varrho_1} \left(H_{\mu\lambda}^{ab} H_{\nu}^{\lambda}_{ab} - \frac{1}{4} g_{\mu\nu} H_{\lambda\sigma}^{ab} H^{\lambda\sigma}_{ab} \right) \\ &\quad + \frac{1}{\varrho_2} \left(H_{\mu\lambda}^{ab} H_{\nu}^{\lambda cd} - \frac{1}{4} g_{\mu\nu} H_{\lambda\sigma}^{ab} H^{\lambda\sigma cd} \right) \epsilon_{abcd} \end{aligned} \quad (2.19)$$

The field equations for ϖ differ slightly from the standard Yang-Mills form and read

$$\nabla^\mu \left(\frac{1}{\varrho_1} H_{\mu\nu}^{ab} + \frac{1}{\varrho_2} \epsilon^{abcd} H_{\mu\nu cd} \right) = 0, \quad (2.20)$$

where ∇ is the covariant derivative of the metric constructed from the tetrad e_μ^a (as a result of field equations for the action (2.17) the spacetime torsion vanishes) acting on spacetime indices, and of connection ϖ acting on the algebra indices.

Unfortunately the gauge theory with a non-compact gauge group is known to suffer from the unitarity problem at the quantum level [14]. Moreover, already in the classical case their coupling to gravity is problematic because the energy density is not positive definite, as can be seen explicitly by inspecting the T_{00} component of the energy-momentum tensor in (2.19) above. We will discuss this in more details in Sect. IV while investigating the properties of simple cosmological solutions of the theory defined by the action (2.17).

III. BI-METRIC INTERPRETATION

In this section we briefly discuss the relation between the gauged Maxwell theory and bi-metric theories. To begin with let us notice that the gauge transformations of the connection ϖ introduced in the preceding section have, formally, exactly the same form as the ones of the gravitational connection ω . Therefore we can associate with it another “tetrad” field one form f^a and the “torsion” two-form τ^a

$$df^a + \varpi^a_b \wedge f^b = \tau^a. \quad (3.1)$$

Having defined the new tetrad field f^a it is possible to construct yet another gauge invariant action, which is exactly the Einstein action, but this time for the fields f and ϖ . Indeed

$$S_{E'} = \frac{1}{64\pi G'} \int \epsilon_{abcd} \left(H_{\mu\nu}{}^{ab} f_\rho^c f_\sigma^d - \frac{\Lambda'}{3} f_\mu^a f_\nu^b f_\rho^c f_\sigma^d \right) \epsilon^{\mu\nu\rho\sigma} \quad (3.2)$$

is manifestly gauge invariant, exactly as the Einstein action (2.14) is. The theory defined by the sum of the actions S_E and $S_{E'}$ reminds very much the $f - g$ theory of Isham, Salam, and Strathdee [15] (for recent discussion and references see e.g., [16]) in the first order formalism. The only part that is missing is the contact term between tetrads e and f . However, since the tetrads e and f transform under different gauge groups it is only possible to couple the metrics constructed out of them $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$ and $q_{\mu\nu} = f_\mu^a f_\nu^b \eta_{ab}$. In the usual $f - g$ theory setup such constant terms break the $(diff)^2$ invariance of the action $S_E + S_{E'}$ down to its diagonal subgroup. This is however not the case here: our theory from the very start has just one built-in diffeomorphism invariance that results from the gauged translations. Therefore adding the contact term does not lead to breaking of any symmetries of our theory. The most general contact term has the form

$$S_{cont} = M^2 (-g)^u (-q)^{(1/2-u)} V(g^{-1}q) \quad (3.3)$$

where $(g^{-1}q)^\mu{}_\nu = g^{\mu\rho} q_{\rho\nu}$ and V is an arbitrary scalar potential that can be constructed from this tensor.

The $f - g$ theories have been investigated recently from many perspectives, e.g., in [16], [17], [18], [19], [20], [21]; see also [22] for a recent comprehensive review. We leave it to the future work to check if in any of these perspective (brane worlds, Kaluza-Klein models, noncommutative geometry, ...) the local Maxwell symmetry can find its natural habitat.

IV. EXAMPLE: A SIMPLE COSMOLOGICAL MODEL

In this section we consider, as an example, a simple cosmological solution of the model described by the action (2.17). In the course of this exercise we will encounter

the problem of non-positively defined energy, mentioned in Sect. II.

We make use of the simplest, flat FRW metric, which has the form

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2) \quad (4.1)$$

In what follows we analyze the dynamics of this model using as a starting point the variational principle, not the equations of motion, and therefore we substitute the metric (4.1) into the Einstein action (2.14) and neglecting the (infinite) space volume prefactor we obtain

$$S_E = \frac{1}{8\pi G} \int dt \left(-\frac{3}{N} a \dot{a}^2 - \Lambda a^3 N \right). \quad (4.2)$$

Let us now turn to the gauge fields part of the action. The first step is to find the most general form of the fields, consistent with the background symmetries of the FRW spacetime (4.1). The desired form is easy to guess. Indeed the only objects that could be present are tensors invariant under action of the group of space symmetries, which in this case is just the Euclidean group. Therefore we take the only non-vanishing components of the connection ϖ to be

$$\varpi_i^{0\alpha} = f(t) \delta_i^\alpha, \quad \varpi_i^{\alpha\beta} = g(t) \epsilon_i^{\alpha\beta}, \quad \alpha, \beta = 1, 2, 3. \quad (4.3)$$

In the Appendix we prove that this ansatz is indeed correct.

Substituting this and the form of the metric (4.1) to the actions S_1 in (2.15), we get

$$S_1 = \frac{1}{\varrho_1} \int dt \left(\frac{3a}{N} (\dot{g}^2 - \dot{f}^2) + \frac{3N}{a} g^2 (4f^2 - g^2) \right) \quad (4.4)$$

Similarly, the action S_2 in (2.17) takes the form

$$S_2 = \frac{1}{\varrho_2} \int dt \left(\frac{12a}{N} \dot{f} \dot{g} - \frac{24N}{a} f g^3 \right) \quad (4.5)$$

We see that in the action (4.4) the sign of the kinetic term is wrong and the potential is not bounded from below, which, as discussed above, exhibit the problems arising in the case of a non-compact gauge group.

The Einstein equations can be obtained from the action $S_E + S_1 + S_2$ by varying over the scale factor $a(t)$ and the lapse function $N(t)$. Since the latter serves as a Lagrange multiplier, in the resulting equation one can set $N(t) = 1$. As a result we get

$$\begin{aligned} \left(\frac{\dot{a}}{a} \right)^2 - \frac{\Lambda}{3} &= \frac{8\pi G}{3\varrho_1} \left[\frac{3}{a^2} (\dot{g}^2 - \dot{f}^2) - \frac{3}{a^4} g^2 (4f^2 - g^2) \right] \\ &+ \frac{8\pi G}{3\varrho_2} \left[\frac{12}{a^2} \dot{f} \dot{g} + \frac{24}{a^4} f g^3 \right]. \end{aligned} \quad (4.6)$$

and

$$\begin{aligned} \frac{2\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 - \Lambda = & -\frac{8\pi G}{3\varrho_1} \left[\frac{3}{a^2} (\dot{g}^2 - \dot{f}^2) \right. \\ & \left. - \frac{3}{a^4} g^2 (4f^2 - g^2) \right] - \frac{8\pi G}{3\varrho_2} \left[\frac{12}{a^2} \dot{f}\dot{g} + \frac{24}{a^4} f g^3 \right] \end{aligned} \quad (4.7)$$

Comparing these equations with the Friedmann equations in the canonical form

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{\Lambda}{3} = \frac{8\pi G}{3}\varepsilon, \quad \ddot{a} - \frac{\Lambda a}{3} = -\frac{4\pi G}{3}(\varepsilon + 3p)a$$

we see that in our case the energy density is

$$\begin{aligned} \varepsilon = & \frac{1}{\varrho_1} \left[\frac{3}{a^2} (\dot{g}^2 - \dot{f}^2) - \frac{3}{a^4} g^2 (4f^2 - g^2) \right] \\ & + \frac{1}{\varrho_2} \left[\frac{12}{a^2} \dot{f}\dot{g} + \frac{24}{a^4} f g^3 \right] \end{aligned} \quad (4.8)$$

and clearly is not positive definite. As for the pressure,

$$p = \frac{\varepsilon}{3} \quad (4.9)$$

as it should be, because the field ϖ is a massless Yang-Mills field. Therefore eqs. (4.6), (4.7) describe a universe with cosmological constant filled with radiation. Therefore the solutions are

$$\begin{aligned} a(t) &= C\sqrt{2(t-t_0)}, & \Lambda &= 0 \\ a(t) &= C\sqrt{\cos\left(\sqrt{\frac{|\Lambda|}{3}}(2(t-t_0))\right)}, & \Lambda &< 0 \\ a(t) &= C\sqrt{\cosh\left(\sqrt{\frac{|\Lambda|}{3}}(2(t-t_0))\right)}, & \Lambda &> 0. \end{aligned}$$

Notice that the form of the first Friedmann equation (4.6) forces the energy density to be strictly positive or positive definite (for negative and vanishing Λ) or bounded from below (for positive Λ). Therefore the coupling to the gravitational field seems to exclude some possible configurations of the field ϖ with large negative energy density. It would be interesting to see if this is a consequence of the cosmological ansatz (4.1), or a somehow generic result.

For completeness we write the g and f equations of motion. They read

$$\frac{1}{\varrho_1} \left[-(a\dot{g}) + \frac{4}{a} g f^2 - \frac{2}{a} g^3 \right] - \frac{1}{\varrho_2} \left[(2a\dot{f}) + \frac{12}{a} f g^2 \right] = 0 \quad (4.10)$$

and

$$\frac{1}{\varrho_1} \left[(a\dot{f}) + \frac{4}{a} g^2 f \right] - \frac{1}{\varrho_2} \left[(2a\dot{g}) + \frac{4}{a} g^3 \right] = 0 \quad (4.11)$$

It is worth noticing that these equations are related to the continuity equation

$$\dot{\varepsilon} + 3H(\varepsilon + p) = \dot{\varepsilon} + 4H\varepsilon = 0, \quad (4.12)$$

where we used (4.9). Indeed substituting (4.8) to (4.12) one finds

$$\dot{f} \boxed{(4.10)} + \dot{g} \boxed{(4.11)} = 0. \quad (4.13)$$

For the simplest possible solution of these equations, with f and g being time-independent constants, excluding the trivial possibility $g = 0$, we find from (4.11) that

$$f = \frac{\varrho_1}{\varrho_2} g.$$

Eq. (4.10) tells that a nontrivial solution is possible only if

$$\left(\frac{\varrho_1}{\varrho_2}\right)^2 = -\frac{1}{4},$$

so that the solution does not exist. Similar conclusion can be reached if one takes $\dot{g}, \dot{f} \sim a^{-1}$. This suggests that the dynamics of the fields f and g is quite nontrivial and will be discussed elsewhere.

Appendix: Cosmological gauge field

In this appendix we derive the most general form of the gauge field h_μ^{ab} on the flat FRW background. The infinitesimal symmetries of this background are spacial translations and rotations, forming together the three dimensional Euclidean group with the algebra (since the metric on the constant time section of the flat FRW spacetime is, up to the rescaling, just δ_{ij} , we will be not very careful in placing indices up or down)

$$[R_i, R_j] = \epsilon_{ijk} R_k, [R_i, T_j] = \epsilon_{ijk} T_k, [T_i, T_j] = 0. \quad (A.1)$$

The Killing vector fields of the flat FRW spacetime form a representation of this algebra

$$\begin{aligned} T_i &= \frac{\partial}{\partial x^i} \\ R_i &= \epsilon_{ijk} x^j \frac{\partial}{\partial x^k} \end{aligned} \quad (A.2)$$

The gauge field invariance under the action of symmetries of spacetime means that the Lie derivatives of the field along the Killing vectors of the symmetries are zero, up to the gauge transformations, i.e. (for general theory see [23] and [24])

$$\mathcal{L}_{T_i} h_\mu^{ab} = \partial_\mu \lambda_i^{(T)ab} + [h_\mu, \lambda_i^{(T)}]^{ab}, \quad (A.3)$$

$$\mathcal{L}_{R_i} h_\mu^{ab} = \partial_\mu \lambda_i^{(R)ab} + [h_\mu, \lambda_i^{(R)}]^{ab}. \quad (A.4)$$

There are important integrability conditions for these equations resulting from the identity

$$[\mathcal{L}_X, \mathcal{L}_Y] = \mathcal{L}_{[X, Y]},$$

which leads to the following condition on the gauge parameters λ

$$\mathcal{L}_X \lambda^{(Y)} - \mathcal{L}_Y \lambda^{(X)} - [\lambda^{(X)}, \lambda^{(Y)}] = \lambda^{([X, Y])}. \quad (\text{A.5})$$

In the case of two translations equation (A.5) takes the form

$$\partial_i \lambda_j^{(T)ab} - \partial_j \lambda_i^{(T)ab} - [\lambda_i^{(T)}, \lambda_j^{(T)}]^{ab} = 0. \quad (\text{A.6})$$

This equation tells that the “field strengths” of the “gauge field” $\lambda_i^{(T)ab}$ vanishes, and since the equation (A.3) is gauge-covariant, it follows that $\lambda_j^{(T)ab}$ must vanish itself.

Then the equation (A.5) applied to rotations and translations reduces to the condition

$$\partial_i \lambda_j^{(R)ab} = 0$$

and therefore $\lambda_j^{(R)ab}$ is just a constant. Then the equation (A.5) applied to rotations takes the form

$$[\lambda_i^{(R)}, \lambda_j^{(R)}]^{ab} = \epsilon_{ijk} \lambda_k^{(R)ab}, \quad (\text{A.7})$$

which means that the matrices $\lambda_i^{(R)ab}$ form a representation of the rotation group $\text{SO}(3)$.

Knowing this we can now return to the equations (A.3), (A.4) which now take a much simpler form

$$\mathcal{L}_{T_i} h_\mu^{ab} = 0 \quad (\text{A.8})$$

$$\mathcal{L}_{R_i} h_\mu^{ab} = [h_\mu, \lambda_i^{(R)}]^{ab} \quad (\text{A.9})$$

Equation (A.8) tells that the gauge field depends only on time $h_\mu^{ab}(x, t) = h_\mu^{ab}(t)$. Then from (A.9) it follows that h_0^{ab} commutes with all the matrices $\lambda_i^{(R)}$ and therefore must be zero, and

$$\epsilon_{ijk} h_j^{ab} = [h_k, \lambda_i^{(R)}]^{ab} \quad (\text{A.10})$$

Returning to eq. (A.7) we see that the matrices $\lambda_i^{(R)ab}$ could be represented as matrices of generators of rotations in the Lorentz group $\text{SO}(3, 1)$ so that explicitly

$$\lambda_i^{(R)\alpha\beta} = \epsilon_i^{\alpha\beta}, \quad \lambda_i^{(R)\alpha 0} = 0, \quad \alpha, \beta = 1, 2, 3 \quad (\text{A.11})$$

Substituting this in (A.10) one can easily check that there are two solutions

$$h_j^{0\beta} = f(t) \delta_j^\beta, \quad h_j^{\alpha\beta} = g(t) \epsilon_i^{\alpha\beta} \quad (\text{A.12})$$

which is the ansatz used in the main text.

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